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THE EXTRACURRICULAR WORK IN MATHEMATICS IN THE PRIMARY AND LOWER SECONDARY SCHOOL STAGE – AN IMPORTANT FACTOR FOR DISCOVERING AND DEVELOPING MATHEMATICAL TALENT

Abstract
of the PhD Thesis

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Contents

Introduction

As a long-time teacher at Sofia Mathematical School, my work is closely related to the in-depth development of the mathematical abilities of gifted students from the elementary school grades. The stated willingness of the Ministry of Education and Sciences to deepen the knowledge of mathematics and technology in connection with Bulgaria's participation in the STEM coalition is the incentive that encouraged me to write this dissertation.

The aim of this dissertation is to show that the development of mathematical talent should start from the last grades of the primary stage or at the latest in the lower secondary stage of education. This provides an opportunity for full development of talent and its realization in secondary school, and later in university and in life in general.

The actuality of the chosen topic is determined by the need to:

- ✓ to develop the skills, abilities and potential of students in the field of mathematics, informatics and technology;
- ✓ to discover, develop and support mathematical talents and to help their realization and personal development;
- ✓ to form and develop students' needs for independent development of competences essential for realization in life.

Hypothesis – Early discovery and purposeful development of mathematical talent is a prerequisite for its successful realization.

The emphasis is on the discovery, development and realization of mathematical talent.

The object of the study is extracurricular work (organization, amount, content and methods) in mathematics with students from 8 to 14 years of age.

The subject of the research is creating interest in extracurricular activities in mathematics, methods of formation of cognitive skills, discovering, developing and training mathematical talents.

To achieve the set goal, the following **tasks** were completed:

1. Study of methodical, pedagogical and psychological literature of established authors, with a focus on development and realization of talents, mathematical talents and competition mathematics.

2. Analyzing and synthesizing the studied ideas and proposing a model for the realization of training, in which the mathematical talent is maximally developed and is effectively implemented in school and after graduation from secondary education.

3. Development of methodological materials for extracurricular work in mathematics.

4. Development of a methodology for evaluating individual students when working in a team (for example, in team competitions).

5. Conducting a survey among students and parents regarding the impact of learning in extracurricular activities in mathematics and their implementation in various mathematical competitions and in life.

The methods used in the study are:

1. Study of methodical, pedagogical and psychological literature of established authors on the researched problem.

2. Analysis and synthesis of the theoretical study of the posed problem and the build of the author's concept.

3. Pedagogical observation.

4. Questionnaire survey.

5. Statistical processing of survey data.

The dissertation consists of:

1. *Introduction*.
2. *The role of extracurricular training and competitions for discovering and developing mathematical talent.*
3. *Analysis of mathematics curricula in elementary and lower secondary school.*
4. *Pedagogical experiment of the research.*
5. *Systems of tasks.*
6. *Assessment of mathematical talent.*
7. *Conclusion.*
8. *References.*
9. *Applications*.

By looking at the specificity of working with young mathematical talents, we come to the conclusion that extracurricular work, started in primary and junior high school, is essential for the development and realization of students. Properly selected topics for extracurricular work, considered in development in several classes, provide an opportunity for a more complete, deep and lasting mastering of knowledge. The selection of tasks with increasing difficulty, as well as non-standard tasks, provoking creative thinking is essential for creating successful and realized young talents. The opportunity to participate in competitions is a strong additional motivating factor for developing mathematical talent.

CHAPTER 1. The role of extracurricular training and competitions for discovering and developing mathematical talent

Mathematical talent - what is it?

What is talent and who can be called talented? Is there a difference between a talented, gifted and outstanding student? Gifted are children whose mental development is above average and whose abilities develop so rapidly that they need and benefit from specifically selected educational programs. Tabov (2004, p. 31) emphasizes that talent can be interpreted as *creative giftedness* and prefers to talk about *outstanding students* – those who show a combination of natural passive aptitudes and active ones in the form of striving for their realization through participation in various competitions.

Talent is like a nugget of gold. First, we have to make an effort to find it, sifting through a lot of sand. Then we have to process and polish it to make it shine. Mathematical talent should also be sought, and among young students. They are not yet taught clichés, trained to perform procedures, and not intimidated that math is scary. At this age, it is easy to find out who has a mathematical aptitude, because the formal knowledge acquired in school is still little, and finding the solution to a more difficult case requires mostly logical thinking.

How to recognize (discover) talent?

Most parents consider their child gifted, but there is a difference between a high achiever in school and a child who is intellectually gifted. Discovering talent in the early school years is not always an easy task. Gifted children have special traits that can help identify them among other students. A gifted child must possess a combination of intellectual, emotional and psychological qualities. Most often, the child's qualities are first noticed and developed in the family. There is a general consensus in early childhood development research that this period is key to a child's intellectual development. “A child's future abilities and performance are strongly influenced by his preschool home environment” (Serebryakov & Langer, 1999, p. 91). Very often, the family environment is not sufficient to develop and support a child's mathematical talent. A child's fast, flexible and non-standard thinking is sometimes frightening even for elementary teachers who do not have a mathematical background. Wise teachers direct these children to math competitions where talent is tested and possibly specialized training is provided.

There are several mathematics competitions for primary school students in Bulgaria. Even for the 1st grade, there are international (European Kangaroo, Mathematics without Borders, Beaver), national (Ivan Salabashev Mathematics Tournament, Chernorizets Hrabar, Viva Mathematics with Computer) and regional mathematics competitions (Sofia Mathematics Tournament, Christmas and Easter Mathematics Competition). These events are the best places where talented and gifted students can be discovered. The results of the competition, as a means of comparison with the achievements of other participants, make it possible to make a clearer assessment of the child's abilities (Tsvetkova, 2016). The system of mathematics competitions during children's primary school age helps to discover talent and is a stimulating and motivating element for its development.

Developing talent and the role of mathematical competitions in its manifestation

A high IQ alone is not a guarantee of success for a gifted child. Ambition, energy and determination, as well as family and social environment, play a key role. Highly intelligent children may have the same intellectual ability, but they are extremely different in personality, interests and achievements. The teacher's role is not only to educate the gifted child academically, but also to strive to provide him with a suitable environment so that he can to develop his talent. First, he must be helped to study and learn effectively. Then to be directed towards appropriate goals and given the opportunity to check his progress (Marland, 1971). One of the things that most motivates students to devote a lot of time to mathematics is undoubtedly the opportunity to participate in national and international competitions. On these they can measure their achievements and progress.

Problems as a tool for developing mathematical talent

One possible goal for the manifestation of mathematical talent is excellent performance in mathematical competitions, and the way to achieve it is systematic training. In it, the problems come to the fore. Especially valuable are topics, tasks of which make it possible to consider the topic in several consecutive years, consolidating what has been learned and then building on it. One such topic is Arithmetic Puzzles. Solving them only requires knowledge of how to perform arithmetic operations, which makes the topic suitable for different age groups. At the same time, puzzles can be composed that require many and varied considerations to find their solution. Also, the fact that there are puzzles that have more than one solution or no solution presents young students with problems unknown to them until now, since the tasks they solve in school usually have only one correct answer. In order to cope with solving puzzles, children are given a system of principles (rules) to help them. Some of these principles correspond to the properties of arithmetic operations and numbers. Since the approaches to solving puzzles can be different, in the process of work, students see that a solution to a problem can be reached by different paths, some of which are shorter or more elegant than others. This shows the richness of mathematics, stimulates curiosity and creativity in children. Adding the satisfaction and pride that young students feel after solving a difficult task, for which even parents are often unable to help, we can say that the topic "Puzzles" is extremely useful for the development of young mathematicians.

CHAPTER 2. Analysis of mathematics curricula in elementary and lower secondary school

New emphases are becoming leading in the educational world: literacy – reading, mathematics, science; key competencies; active learning in a constructivist environment (problem-based approaches); lifelong learning, including non-formal (non-institutional); independent learning and reflection skills; interdisciplinary of training.

There are, undoubtedly, basic knowledges that must be learned and mastered to give us the freedom to use them effectively in situations where much more than their direct application is required. Such is the knowledge of arithmetic, which is mainly studied in the primary stage.

In a primary stage, 46-50% of the time is devoted to teaching new knowledge and the same amount to consolidating it, while in a lower secondary school, the time devoted to new knowledge is 60%, and 32% is for exercises.

There is an imbalance between the time devoted to exercise and new knowledge, especially during the transition from primary to lower secondary school. For 4-th grade students, 50% of the time is for exercises, while the next year, 5-th grade students have only 32% for them.

Perhaps, these facts are the key to the widespread interest of primary school students in mathematics competitions and extracurricular work of math offered by various schools. Students meet there another face of mathematics – fun, provocative, non-standard. They see that there are math problems in which we calculate nothing or almost nothing. Children notice that understanding the task is almost as important as coming up with and describing the solution, and that there may be more than one solution. They face the need to think deeply and for a long time about a problem.

For more than 20 years, extracurricular mathematics classes have been held at Sofia Mathematical School for 2-nd, 3-rd and 4-th grade students. In the course of these years, themes were formed that are suitable for the respective class. Part of the topics were formed in two books: “First Mathematical Reader” and “Mathematical Reader”, and later expanded and supplemented in the NEMO books for 2-nd, 3-rd and 4-th grades. These teaching materials are also used by other schools and colleges. Some of the topics that are developed in these aids and on which second, third and fourth grade students are prepared in extracurricular activities led by Sofia Mathematical School teachers are the following:

1. Logical problems.
2. Overflow Tasks.
3. Operations with natural numbers, application of properties of parity.
4. Application of the rules for operations with natural numbers.
5. Modeling word problems with Euler circles.
6. Dirichlet's principle.
7. Inversion method or back-to-front problem solving – with one, two or three chains.
8. Measurement, units of measurement, conversion from one unit of to another.
9. Circumferences and area of basic geometric figures and combinations with them.

10. Problems for weighing balance scales and exchanging objects of equal value.

11. Counting of possibilities – combinatorial problems solved with counting or graph-tree.

12. Clock problems – adding and subtracting hours and minutes, counting digits in time notation problems.

13. Calendar tasks – specify a day of the week for a past or future date.

14. Modeling word problems using segments.

15. Modeling word problems with elementary systems of equations without using formal notation, or in an arithmetic way.

16. "Invented" arithmetic operations.

17. Purchase problems – modeling word problems with elementary systems of equations without using formal notation.

18. Motion problems.

19. Triangle inequality.

20. Counting digits and numbers.

21. Arithmetic puzzles.

22. Gaussian summation.

23. Diophantine equations.

Some of the topics studied in schools from 2-nd to 4-th grade continue to be studied in lower secondary school. There are also topics that build on what has been learned in the compulsory curriculum lessons. The aim is to expand knowledge and develop problem-solving skills through analysis and synthesis, inductive reasoning, using the method of assuming the opposite and logical reasoning. The skills that students develop at this stage of their studies are extremely useful for their further development as mathematicians and mathematical competitors. The experience they gain in solving hard problems with relatively little knowledge of mathematics helps them in their further development as mathematicians when they have to solve open-ended problems.

CHAPTER 3. Pedagogical experiment of the research

The research in this thesis can be classified as applied, according to its problem orientation, empirical, according to the subject of the research activity, and explanatory, according to its purpose. This leads to the selection of a research approach that fits these characteristics. Due to the impossibility to collect large amounts of standardized information, to use a deductive approach to knowledge, and the existence of a very close relationship between the researcher and the researched objects, it is appropriate to apply the qualitative research approach.

The pedagogical observation is conducted on the students attending a school for extracurricular work in mathematics, from three graduating classes who completed the 12-th grade in the years 2004, 2012 and 2020 respectively. I have worked with all of them from the 5-th to the 12-th grade in Sofia Mathematical School and before their admission to the school during their attendant in the schools for extracurricular training. The students who actively participated in the school during the pedagogical experiment took part in national competitions in mathematics and were among the prize winners, and some of them also won prizes in international competitions.

A common trait of all successful mathematics competitors that they started their activities with extracurricular and competitive mathematics in elementary school or at the latest from the beginning of lower secondary school. All of them have participated in math competitions – mainly the National Mathematical Olympiad, but also many other national and regional competitions. In the high school stage, they all developed as talented and successful competitors, not only in mathematics, but also in related disciplines, the basis of which is knowledge of mathematics – physics, informatics, mathematical linguistics, chemistry, astronomy. All of them continue (some have already completed) their education in the best universities in Bulgaria and in the world and have outstanding success, including in the scientific field.

I have expanded these observations of mine with an anonymous survey that I asked former students of the Sofia Mathematical School to complete. From the answers to the first question in the survey, it is clear that only 3.9% of all respondents started with competition mathematics only after entering high school. All the rest started their activities earlier, with the largest number of those who started activities between the ages of 9 and 11 – 63%.

Of all those who completed the survey, 88% indicated that they had participated in math competitions in elementary or lower secondary school.

80% of the respondents say that participation in the extracurricular activities significantly or significantly high influenced their development as professionals and their realization in life.

It can be concluded from the survey that early involvement in extracurricular activities in mathematics and participation in mathematical competitions significantly affects not only educational achievements but also the professional and life realization.

CHAPTER 4. Systems of tasks

This chapter presents two systems of tasks for working with gifted students. The topic of each of them gives the opportunity to be developed in several consecutive grades. Thus, students have the opportunity to reinforce the topic in different years. The tasks are selected in accordance with the age of the students and their knowledge acquired from the compulsory curriculum. Similar systems of task can also be found in: Tsvetkova 2010; Bankov, Tsvetkova 2014 and 2015.

Mathematical puzzles

One of the topics that can be covered in several consecutive grades is “Puzzles”. Solving arithmetic puzzles requires only knowledge of how to perform arithmetic operations, which makes the topic suitable for different age groups. At the same time, puzzles can be composed that require many and varied considerations to find their solution. Also, the fact that there are puzzles that have more than one solution or no solution presents young students with problems unknown to them until now, since the problems they solve in school usually have only one correct answer. This creates the feeling in young and smart children that it is enough to mentally consider or calculate something or to “guess” the answer semi-intuitively in order to claim to have solved the problem. That is why one of the greatest difficulties for the youngest students when solving puzzles is reasoning why this is the only (if any) solution and why we are sure that all the solutions to the puzzle have been found.

To teach solving puzzles, we give children a system of principles (rules) to help them. Their number increases according to the knowledge that the students have. Some of these principles correspond to the properties of arithmetic operations and numbers. We also recommend not to make random checks, but to consider which of the symbols we can uniquely determine. For those we cannot immediately determine, we write down possible values, trying to estimate when we have as few checks as possible. Most often we start from the unit's digit or from the oldest order or from the symbol that occurs most often. When the algorithm requires branching, we teach children to follow all the branches and methodically consider all possibilities. This way, there is no danger of missing a solution, and if the puzzle doesn't have a solution, we can prove it. It is important for students to understand that solving a puzzle means finding *all its solutions* or showing that there are none.

Since the approaches to solving puzzles can be different, in the process of work, students see that a solution to a problem can be reached by different paths, some of which are shorter or more elegant than others. This shows the richness of mathematics, stimulates curiosity and creativity in children. Adding the satisfaction and pride that young students feel after solving a difficult task, for which even parents are often unable to help, we can say that the topic "Puzzles" is extremely useful for the development of young mathematicians.

Back-to-front method of problem solving

Another topic that can be considered from second through fifth grade is the application of the back-to-front problem-solving method in word problems or problems for finding an unknown number. This method uses schemas (we call them "chains") that help keep track of the actions described in the task condition and enable the solution to be found starting from the last action performed and working backwards. Thus, step by step, the initial situation is reached, which is usually sought in the task. This method greatly simplifies the solution of the problem, which can usually also be solved by constructing an equation. The equation, however, is often overwhelming to solve, not only for small, but also for older students. This feature of the method reminds us that we can also use it to solve equations, even though it is not always applicable in this aspect.

We start the topic in second grade with adding and subtraction. Later we increase the difficulty and involve more operations until fifth grade when we include operations with fractions, the concept of part of a number, and percentage.

CHAPTER 5. Assessment of mathematical talent

The problems that students solve in math competitions are often used for measuring the mathematics achievement of gifted students. Very often, the assessment of the mathematical abilities of talented children takes place when they appear in competitions. Then they are evaluated by an independent committee and the evaluation is expressed in points and ranking. This method of assessment is well known and has been used for many years.

Some of the competitions are team competitions or have a team stage. For the team competitions, a grade (ranking) is given to the whole team, but this does not give a good idea of the participation of each individual student in the performance of the overall task. This fact challenged us to look for a way to evaluate the participation of individual team members. Such scoring is useful as feedback for competitors as they participate in team competitions.

The author's method for evaluating individual participants in teamwork is considered in the thesis. The method is applicable for both team competitions and for teamwork in regular school classes. It gives a more complete picture of the team's work, including assessments of the main activities performed by team members from the moment of its creation to the completion of the joint task.

In the context of evaluating the development talent, the following conclusions can be drawn related to the use of the considered method:

1. For students in the upper grades (high secondary school stage), it is more appropriate to evaluate the talent of the strongest competitors based on the individual work of each student. For other competitors, as well as for students who do not have the ambition and opportunity to participate in mathematical competitions, teamwork and the method discussed are more appropriate. This conclusion is based on the conducted survey.

2. For students from the lower grades (primary and lower secondary school stage), talent assessment can be done both with individual tasks and with team work using the considered method. This conclusion is based on the fact that some of the competitions for younger students are either team competitions or have a team stage. Personal observations show that teamwork with young students is preferable both from an emotional point of view and as an opportunity for expression.

Conclusion

This work is the result of the author's long experience in working with students who have mathematical talent. The work with these students is both in-class and extra-curricular. The observations and results are related mainly to extra-curricular training are described here. The focus of the work is extracurricular work with primary and lower secondary school students and its importance for the discovery and development of mathematical talent.

The main thesis of the dissertation is that in order to successfully discover and develop mathematical talent in children, it is necessary to start active extracurricular work at the primary or lower secondary school stage of education. Also, for the development of this talent the main tools are competitive tasks, preparation and participation in mathematics competitions.

Specific contributions of the author are:

1. An analysis of the methods and ways of discovering children with mathematical talent and way of working with them.
2. Development of a program for extracurricular work in mathematics for students grades from 2 to 7 grades.
3. Development of study materials for this program for students of grades from 2 to 7 grades, which are also used by other schools for extracurricular work in mathematics.
4. The methods of working with talented students have been applied in practice with several cohorts of Sofia Mathematical School.
5. The impact of this way of working on the development and manifestation of talents has been studied.
6. Theme-based task systems are presented, which allow to be developed and upgraded for different age groups.
7. 7. Author's methodological developments for teaching these topics are presented.
8. An author's method for evaluating the individual achievements of each student participating in a team is proposed.

The following conclusions can be drawn from the conducted observations and research:

- Early discovery of mathematical talent is essential for its development.
- Development of the talent and its manifestation go hand in hand and mutually support each other.
- Mathematics competitions are a powerful tool for the development of the talent.
- Students who have been engaged in extracurricular work in mathematics and have participated in competitions since the primary or lower secondary school stage of their education have a successful professional and life realization.

These conclusions **confirm the working hypothesis**, namely that the early detection and purposeful development of mathematical talent is a prerequisite for its successful realization.

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Declaration of originality

from Iliana Ivanova Tsvetkova
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In connection with the procedure for acquiring the educational and scientific degree “Doctor” in FMI at SU "St. Kliment Ohridski" and defense of the dissertation work presented by me, I declare:

The results and contributions of the conducted dissertation research, presented in my thesis on the topic The extracurricular work in mathematics in primary and lower secondary school – an important factor for discovering and developing mathematical talent” are original and are not borrowed from research and publications in which I do not have.

Iliana Tsvetkova

Sofia, 2023